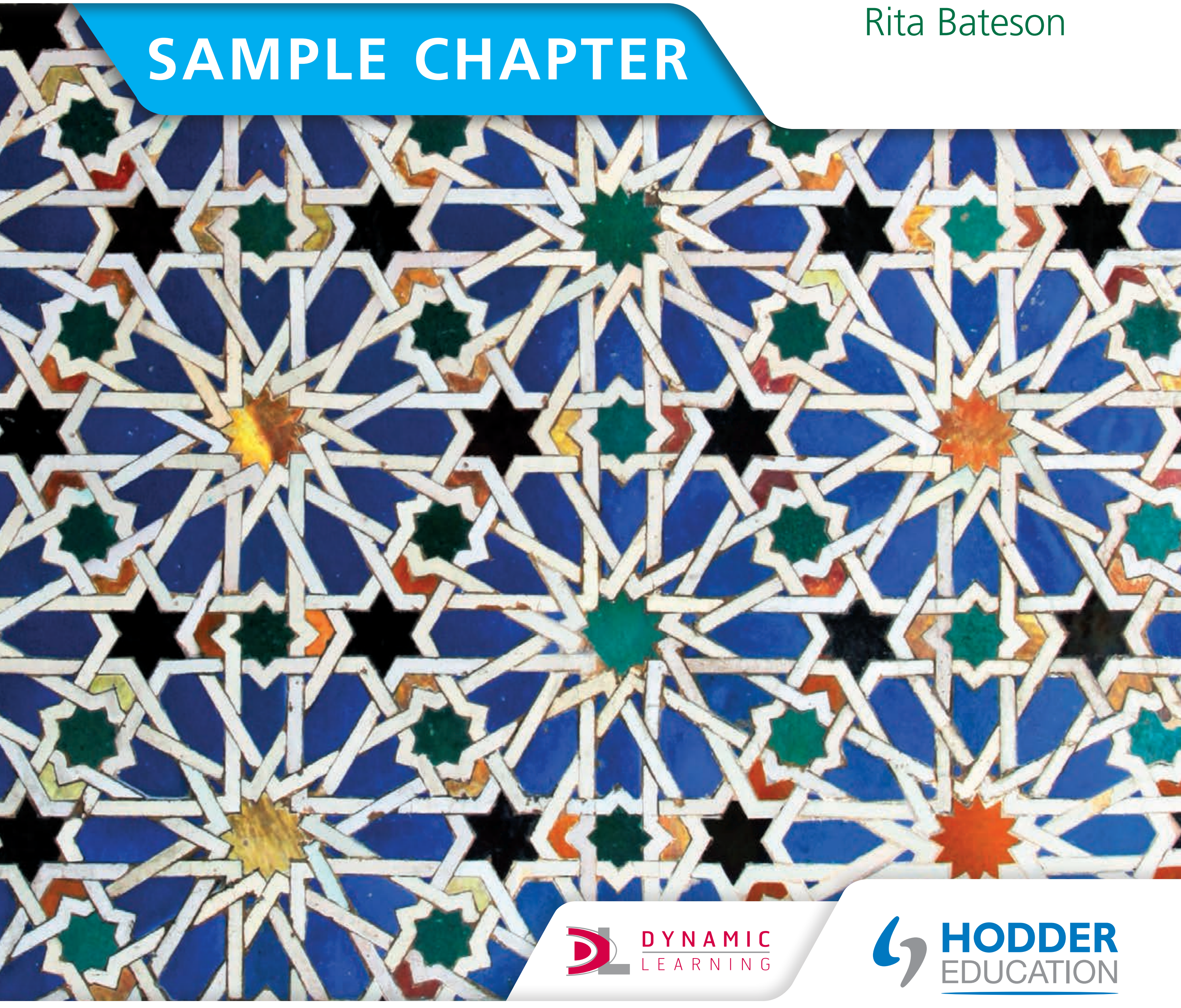
MYP by Concept 4 & 5

Mathematics

Rita Bateson

SAMPLE CHAPTER



DYNAMIC LEARNING

HODDER EDUCATION

Contents

A bright and engaging page design, unlike any other textbook.

Each chapter is framed with a Key concept and a Related concept, and is set in a Global context.

Relationships *Models* *Scientific and technical innovation*

4 How is technical innovation changing our ideas of public and private space?

The Statement of Inquiry provides the framework for the inquiry, and the inquiry questions then lead the exploration as they are developed through each chapter.

Modelling allows us to solve new spatial **relationship** problems arising from **technical innovation**.

Drone photography

CONSIDER THESE QUESTIONS:

Factual: How can we calculate unknown angles and sides? How do trigonometric relationships work?

Conceptual: Where do geometric shapes occur around us? How do their relationships give us insight into the unknown? How do I find a missing angle or side? What can we measure? What can't we measure? Can we calculate what we can't measure?

Debatable: Can anyone 'own' an angle? How much should models affect our understanding of the real world?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

IN THIS CHAPTER WE WILL …

- **Find out** about drones and what they have to do with trigonometry.
- **Explore** Pythagoras' theorem, geometric shapes and trigonometric relationships.
- **Take action** by debating about the viewing angles and heights arising from drone use.

Each chapter covers one of the four branches of mathematics identified in the MYP Mathematics skills framework.

These Approaches to Learning (ATL) skills will be useful …

- Information literacy skills
- Critical-thinking skills
- Collaboration skills
- Creative-thinking skills
- Transfer skills
- Communication skills

We will reflect on this learner profile attribute …

- Principled – we act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

Assessment opportunities in this chapter:

- **Criterion A:** Knowing and understanding
- **Criterion B:** Investigating patterns
- **Criterion C:** Communicating
- **Criterion D:** Applying Mathematics in real-life contexts

PRIOR KNOWLEDGE

You will already know:

- that angles in a triangle sum to 180°
- that Pythagoras' theorem states that the square of the hypotenuse equals the sum of the squares of the other two sides
- how to use Pythagoras' theorem to find missing/unknown sides of right-angled triangles
- how similar triangles relate to one another
- what a Pythagorean triple is, such as 3, 4, 5 or 5, 12, 13.

KEY WORDS

adjacent	inverse
depression	opposite
elevation	ratio

THINK–PUZZLE–EXPLORE

- **What do you think you know about this topic?**
- **What questions do you have?**
- **How can you explore this topic?**

We have incorporated Visible Thinking – ideas, framework, protocol and thinking routines – from Project Zero at the Harvard Graduate School of Education into many of our activities.

WHAT IS A DRONE?

Drones, or UAVs (unmanned aerial vehicles) are aircrafts without a human pilot on-board. These aerial vehicles are often expensive but come in a variety of sizes and are becoming more commonly used and owned by individuals.

Drones can be mounted with cameras or other devices and are often used by individuals, governments and organizations for a variety of uses and applications. They are traditionally used by the military for dangerous missions, but have become increasingly used for policing, surveillance, scientific investigation and aerial filming.

Assessment opportunities are identified.

You might have seen popular videos, items in the news or viral stories as drones become more and more commonplace. The area swept out, covered by or visible to, a drone depends on its height from the earth and the viewing angle of the camera.

https://youtu.be/mxWd5OuoiRQ

DISCUSS

ATL

- Critical-thinking skills: Evaluate evidence and arguments

Now that drones can survey and record areas that previously were hard to see, what does this mean for our privacy? For our safety? For our creativity? For our discovery?

Activities are designed to develop students' Approaches to Learning (ATL) skills.

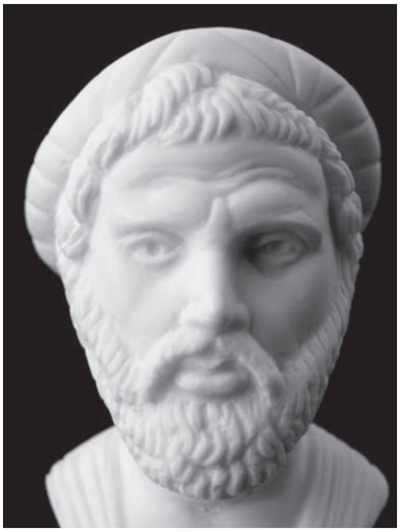
How can we calculate unknown angles and sides?

Geometric shapes are all around us. Throughout time, the relationships shown by these shapes have been used and applied to calculate quantities such as the area of a field, the distance between stars and to navigate the seas. Rules and relationships have been discovered to allow people to calculate what they can't measure.

Triangles in particular have been used throughout the ages as their relationships can help us to model distances and angles to find unknowns. As humankind has moved from the surface of the earth into the skies and the heavens, scientific innovation has brought new opportunities and challenges.

WHO AND WHAT IS PYTHAGORAS?

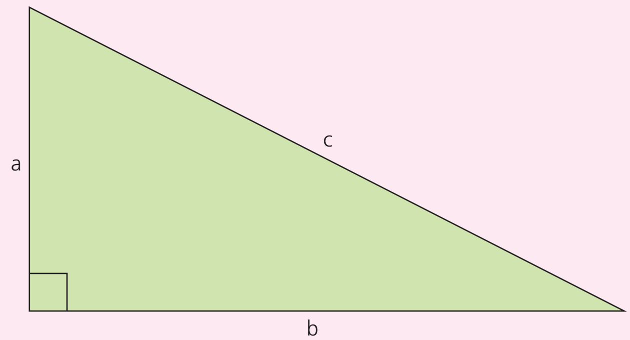
Pythagoras was an ancient Greek philosopher and mathematician who is estimated to have lived between 570 and approximately 495 BCE. The exact details of Pythagoras' life and works have been lost in the mists of time but he is mostly commonly remembered for **Pythagoras' theorem**, which is perhaps one of the oldest and most elegant relationships in geometry.



What is Pythagoras' theorem?

Remember that:

A right-angled triangle contains a right angle. The longest side opposite the right angle is called the **hypotenuse**.



We have looked at Pythagoras in Chapter 5 of *Mathematics for the IB MYP 3: by Concept.* Before we look inside a right-angled triangle to find relationships, we must revise the relationship governing the sides. You will have already learned that the square of the hypotenuse is equal to the sum of the square of the other two sides, or:

$$a^2 = b^2 + c^2 \text{ or } a^2 + b^2 = c^2 \text{ or hyp}^2 = a^2 + b^2$$

For the purposes of consistency, we will use the version of $a^2 + b^2 = c^2$ where a, b are the shorter sides and c is the hypotenuse. It is important to remember that all forms are valid and all will be accepted in assessment and examinations, if the communication is clear, consistent and correct.

Hint

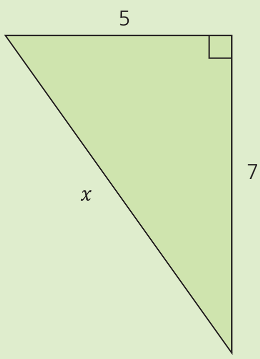
Sometimes it might be faster to write the equation as $c^2 = a^2 + b^2$ if you know you are looking for the hypotenuse.

Example

Problem 1

Calculate the value of x.

Give your answer correct to 1 d.p.



Triangle A

Solution

First you should **state** the theorem:

$c^2 = a^2 + b^2$

Substitute values:

$x^2 = 5^2 + 7^2$

$x^2 = 25 + 49$

$x^2 = 74$

To find x, **inverse** or undo the square:

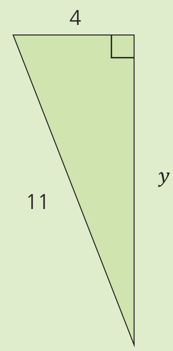
$x = \sqrt{74}$

$x = 8.602325267$ from the calculator

$x = 8.6$ to 1 decimal place

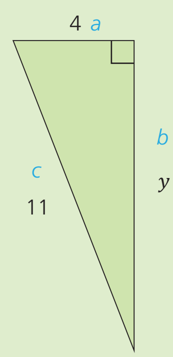
Problem 2

Calculate the length of side y.



Triangle B

Solution



Triangle B with sides labelled

Pythagoras: $c^2 = a^2 + b^2$

BUT we are looking for a shorter side

so $11^2 = 4^2 + y^2$

so $y^2 = 11^2 - 4^2$

$y^2 = 121 - 16$

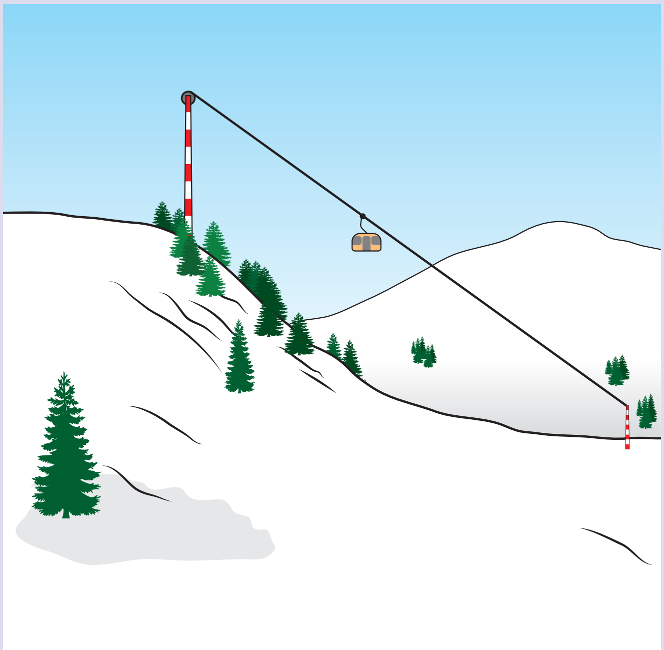
$y^2 = 105$

$y = \sqrt{105}$

$y = 10.24695\ldots$

$y \approx 10.2$

Example

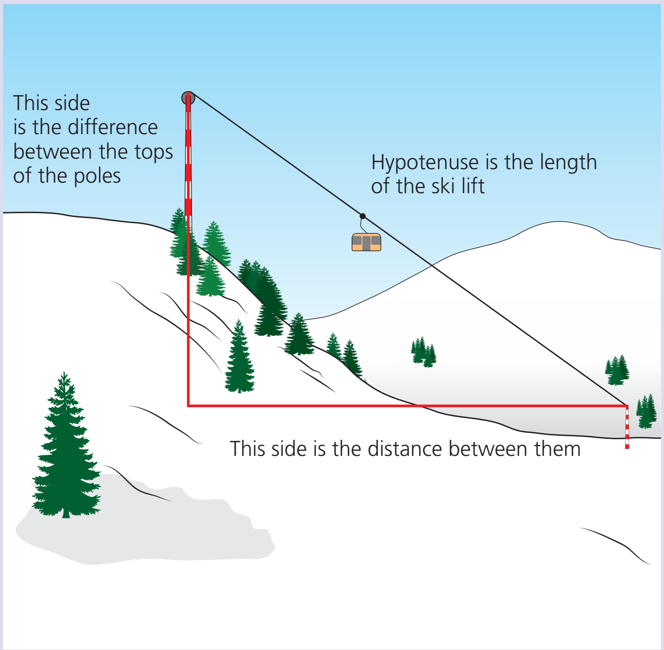


The picture above shows a ski lift with a large and a small pole exactly 370 metres away from each other, as the crow flies (see below). If the top of the small pole is 250 metres directly below the top of the large pole, how long it the ski lift?

▼ Links to: English; Language and literature; Geography

As the crow flies, is an idiom for the shortest distance between two points. Find more **idioms used to describe distance**.

Solution



Worded problems can be made easier by taking out all the unnecessary visual information and finding the triangle.

From the diagram above we can see that

$$c^2 = a^2 + b^2$$

Or $(\text{ski lift})^2 = (\text{height difference})^2 + (\text{distance})^2$

$$(\text{ski lift})^2 = 250^2 + 370^2$$

$$(\text{ski lift})^2 = 62500 + 136900$$

$$= 199400$$

$$\therefore \text{ski lift} = \sqrt{199400}$$

$$= 446.5422712\,\text{m}$$

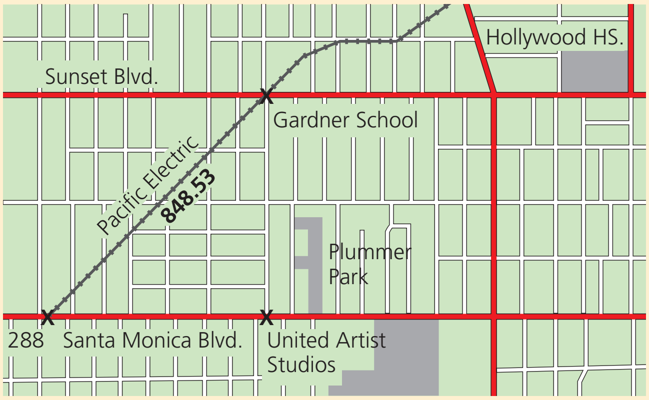
As no level of accuracy was mentioned in the question, an *appropriate* answer would be either 447 m (3 s.f.) or 450 m (same s.f. as the question values).

Example

How far is the United Artist Studios from 288 Santa Monica Boulevard if it is the same distance from the Gardner School?

Hint

Use the Pacific Electric railway line information.



Pacific Electric railway line map

Solution

The Gardner School, the Studios and 288 Santa Monica Boulevard form a right-angled triangle.

We know the two shorter sides are identical as the question tells us they are **equidistant**. This means that we can name each of the sides by the *same* letter, a.

So instead of a and b, Pythagoras' theorem will now look like

$a^2 + a^2 = c^2$

$a^2 + a^2 = (848.53)^2$ as the Pacific Electric railway is the hypotenuse value

$a^2 + a^2 = 720003.16$

$2a^2 = 720003.16$ by collecting like terms

$a^2 = 360001.58$ by halving

$a = \sqrt{360001.58}$ by square rooting

$a = 600.00\,\text{m}$

ACTIVITY: What has a drone got to do with right-angles?

ATL

- Critical-thinking skills: Gather and organize relevant information to formulate an argument

If we consider drones surveying the land from above, where could we see right-angled triangles occurring? How could Pythagoras' theorem help us to calculate important unknowns?

A drone surveying the land from above

Hint

Think of the area covered by a drone's camera.

If we change direction and think about the drones being used to survey upright objects, such as buildings, homes or geographical features, can we find right-angled triangles in these situations also? Consider how this makes you feel.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

How do trigonometric relationships work?

WHAT ARE THE TRIGONOMETRIC RATIOS?

Trigonometry, which comes from the Greek meaning 'triangle measurement' uses the property of similarity to find unknown sides or angles. The fact that side lengths of similar triangles are always in the same ratio has allowed mathematicians to name these ratios and devise uses for them.

These three ratios: **sine**, **cosine** and **tangent** each pair two of the three sides of a right-angles triangle, relative to an angle. The ratios are commonly abbreviated or shortened to sin, cos and tan. See if you can find them on your calculator.

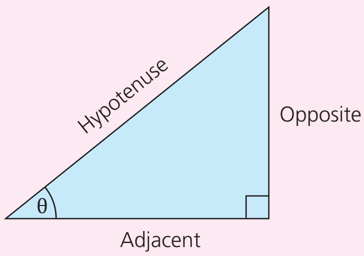
The other two sides of a right-angled triangle are labelled as either opposite or adjacent (touching) depending on/relative to the angle in question.

Links to: Design

Ask your Design teacher about 'innovating the adjacent possible'. What does it mean? How does it relate to the mathematical meaning of adjacent?

Links to: Language and literature; History

Research where the terms **sine**, **cosine** and **tangent** come from. **Investigate** the **history of trigonometry**.



Labelled triangle A

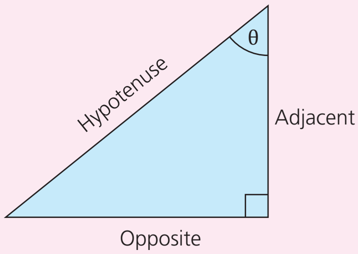
For a right-angled triangle, the sine, cosine and tangent of an angle θ are defined as

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Of course, if the angle moves to a different location, the opposite and adjacent will also move, relative to the angle. The hypotenuse will always be the longest side, opposite to the right angle.



Labelled triangle B

These ratios will always have the same value for any particular angle, no matter what the size of the triangle. This is due to the nature of **similar triangles**.

Until relatively recently, many countries did not allow calculators in examinations and students were required to look up the trigonometric ratios in a book of tables (log book).

As the use of technology to support our mathematics is encouraged in IB, how do we use a scientific calculator or graphical display calculator to find a sin, cos or tan?

Using a scientific or graphical display calculator

All scientific calculators and apps will have buttons for sin, cos and tan.

sin, cos and tan buttons

With some calculators, you press the button first and input the value of the angle second. For others, type the angle value first, then press the sin, cos or tan button.

Test yours now with sin 30° to see which order your calculator prefers. Whichever order gives an answer of 0.5 is the correct one.

Hint

Remember to check mode! If your calculator is not in degree mode (but in radian mode), then your answers will be incorrect for these questions.

ACTIVITY: Inquiry practice

Find the following values on your calculator:

sin 10° =	cos 10° =	tan 10° =
sin 20° =	cos 20° =	tan 20° =
sin 30° =	cos 30° =	tan 30° =
sin 40° =	cos 40° =	tan 40° =
sin 50° =	cos 50° =	tan 50° =

Do you notice any patterns? Can you extend this pattern?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating, and Criterion B: Investigating patterns.

ACTIVITY: How can angles be manipulated?

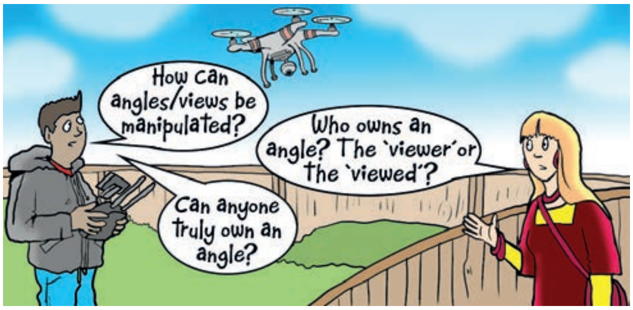
ATL

- Collaboration skills: Listen actively to other perspectives and ideas
- Critical-thinking skills: Gather and organize relevant information to formulate an argument

Two of your friends Robin and Sumaya live next door to one another. Robin has recently purchased a small camera drone and is excited to be using it in his garden. The drone can fly reasonably high and see over fences and trees. He does not fly it beyond the limits of his own garden. He feels that he has a right to fly anywhere in his own garden and that any angles extended by the drone in that space belong to him.

Sumaya lives next door to Robin and is worried about the drone's angles of view. She trusts him not to spy but she also feels that she should have the right to decide where Robin can fly if there is a chance he will invade the privacy of her family. The two cannot agree and have asked you to mediate.

In pairs, or small groups, consider these questions:



Can you 'own' an angle?

EXTENSION

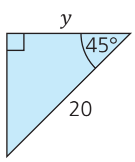
As trigonometric ratios are defined as the ratio of two lengths, what is their unit? Why? Prove this by showing an example or citing a source.

Assessment opportunities

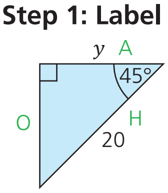
- In this activity you have practised skills that are assessed using Criterion D: Applying Mathematics in real-life contexts.

Can we calculate what we can't measure?

Example 1 Find missing side y



Step 1: Label sides

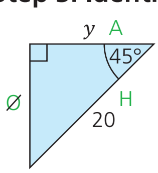


identify the hypotenuse, the side opposite the angle and the touching (adjacent) side

Step 2: Write the ratios

SOH CAH TOA

Step 3: Identify which sides are given/needed



The hypotenuse is given (20)

We are looking for y (adj)

SOH CAH TOA

This is the only one to contain A and H

Step 4: Write formula

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$$

Step 5: Substitute values

$$\cos 45 = \frac{y}{20}$$

Step 6: Re-arrange and **solve**

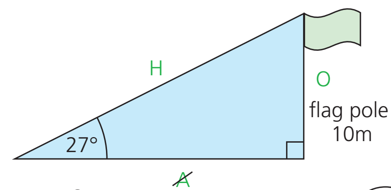
$$\cos 45 = \frac{y}{20}$$

so $y = 20\cos 45$ multiply x20

$y = 14.14$

Example 2

Find the hypotenuse for a right-angled triangle where an observer has an angle of elevation of 27° on a flagpole 10 metres high.



$$\sin = \frac{\text{O}}{\text{H}}$$

$$\sin 27° = \frac{10}{\text{H}}$$

SOH CAH TOA

cross multiply

$\text{H} \sin 27 = 10$

$\text{H} = \dfrac{10}{\sin 27}$

$\text{H} = 22\text{m}$

Using trigonometric ratios

SOH CAH TOA

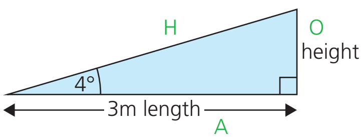
■ A tool for remembering the order of ratios

This is a tool which can be used to help you remember the order of the ratios and how to find them. By laying them out in this way, you can easily choose between the three and it reminds you how to find the **quotient**, i.e. which side is the numerator and which is the denominator.

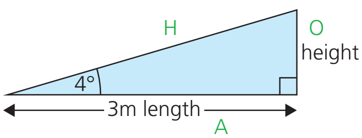
Example 3: worded problem

If a wheelchair ramp has an angle of elevation of 4° and a length of 3m, how high must it be?

First **construct** the triangle using the information provided in the question:



Now **label** sides



SOH CAH TOA

We have A and want O

SOH CAH TOA

$$\tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

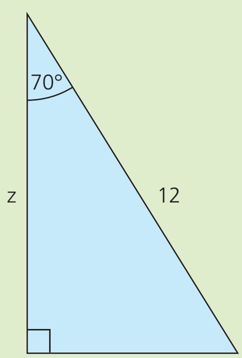
$$\tan 4 = \frac{O}{3}$$

$$= 3 \times \tan 4°$$

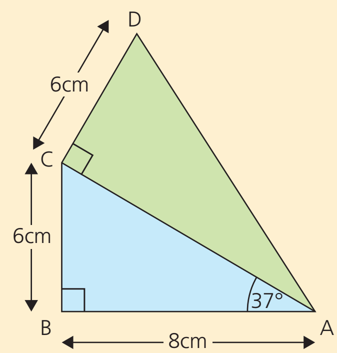
$$= 0.20978$$

The opposite side, or height, is 0.21m high

Find *z*.



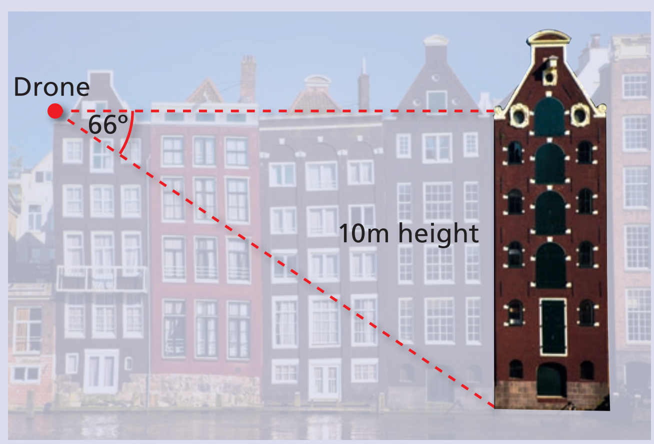
In the diagram below, find all the missing angles and sides using trigonometric ratios and Pythagoras' theorem.



Hint

Don't convert trigonometric ratios until the last step to avoid errors in rounding.

A tourist in Amsterdam wishes to take a photo of a typical canal house using their drone camera. The camera has an angle of depression of 66°.



How far back from the 10m high building should the drone be to photograph *the whole building* as clearly as possible?

How do I find a missing angle or side?

HOW CAN I USE 'SOHCAHTOA' TO FIND ANGLES?

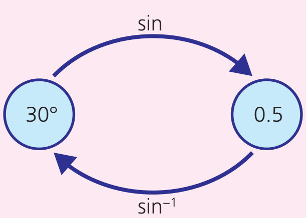
In algebra, you have learned rules for rearranging, transposing or 'manipulating' formulas to find what you are looking for. For example, you may wish to inverse a 'squared' in a Pythagoras problem by square rooting the other side. Trigonometric ratios also have an inverse function each. This allows us to 'undo' and isolate the angle to solve.

$\sin \rightarrow \sin^{-1}$ $\cos \rightarrow \cos^{-1}$ $\tan \rightarrow \tan^{-1}$

To work this out use the $\sin^{-1}$ on the calculator.

$\sin^{-1} 0.5 = 30°$

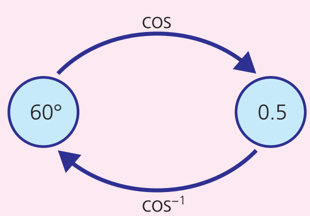
$\sin^{-1}$ is the inverse of sin. It is sometimes called arcsin.

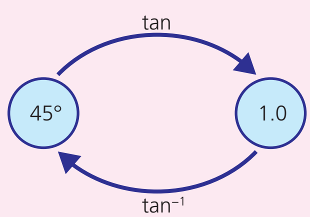


If I know the value of a sin in a given triangle, such as $\sin x = 0.5$, I need to isolate or solve for x to find the angle. Removing the sin, or moving it to the right-hand side, means I must use the inverse $\sin^{-1}$

$\sin^{-1} 0.5 = 30°$

Likewise,





PRACTICE EXERCISES

1 Find the angles, correct to 1 d.p.

a $\sin^{-1} 0.35$

b $\cos^{-1} 0.8760$

c $\tan^{-1} 1.2$

2 Find the angles, correct to the nearest degree

a $\sin^{-1} 0.5$

b $\cos^{-1} 0.176$

c $\tan^{-1} \frac{4}{5}$

Hint

Using a calculator or graphic display calculator

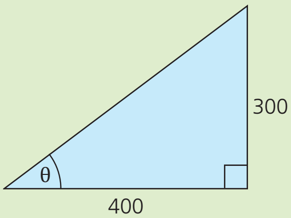
Most apps or calculators have inverse buttons. You may need to use the shift button to access the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ functions.

Ensure that your calculator is in degrees' mode and not radians, this is very important. You will learn more about radians later.

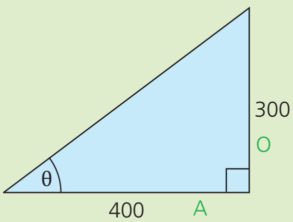
Remember that $\sin^{-1}$ is not a power, it is an inverse operation notation.

HOW DO WE USE THIS INVERSE IN SOLVING PROBLEMS?

Example 1



As before, **identify** the sides.



So we will use $\cancel{S}^{O}_{H}\,\cancel{C}^{A}_{H}\,T^{O}_{A}$

$$\tan = \frac{O}{A}$$

$$\tan\theta = \frac{300}{400}$$

$$\tan\theta = 0.75$$

But we're looking for θ

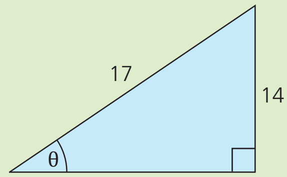
$$\tan\theta = 0.75$$

$$\tan^{-1}$$

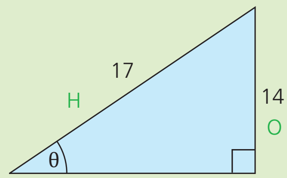
$$\theta = \tan^{-1} 0.75$$

$$\theta = 37°$$

Example 2



Label sides and choose $S^{O}_{H}C^{A}_{H}T^{O}_{A}$



$S^{O}_{H}\,\cancel{C}^{A}_{H}\,\cancel{T}^{O}_{A}$

$$\sin = \frac{O}{H}$$

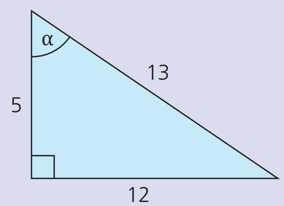
$$\sin\theta = \frac{14}{17}$$

$$\theta = \sin^{-1}\left(\frac{14}{17}\right)$$

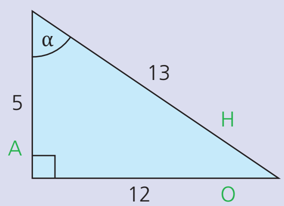
From calculator →

$$\theta = 55.4°$$

Find α



Label sides and choose $S_H^O C_H^A T_A^O$



Unusually, we have all three sides, so we can choose *any* of the ratios.

$$\sin = \frac{O}{H}$$

$$\sin \alpha = \frac{12}{13}$$

$$\sin \alpha = 0.923$$

$$\alpha = \sin^{-1} 0.923$$

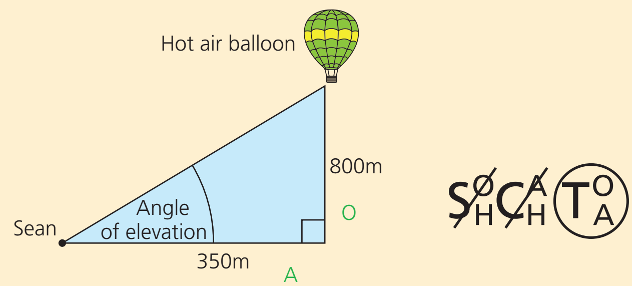
$$\alpha = 67.38013505\ldots$$

$$\therefore \alpha \approx 67°$$

A hot air balloon is flying at a height of 800 metres. If Sean is looking at the balloon 350 metres away from the balloon, what is the angle of elevation through which Sean is looking?

Solution

First **identify** the relevant information from the question and **construct** a triangle to label.



$$\tan = \frac{O}{A}$$

$$\tan \theta = \frac{800}{350}$$

$$\theta = \tan^{-1}\left(\frac{800}{350}\right)$$

$$\theta \approx 66.4°$$

Therefore, Sean must be looking up at an angle of elevation of approximately 66°.

EXTENDED

If Lola is in the hot air balloon and looks down to see Sean, through which angle of depression must she be looking?

The angle of depression is the angle looking down from the balloon. By using the fact that the angles in a triangle add to 180° and the fact that we know the other angles are 67° and 90°, that means

angle of depression = 180° – 67° – 90°

Lola is looking down through (or has an angle of depression of) an angle of 23°.

ACTIVITY: Sundials

ATL

- Creative-thinking skills: Design new machines and technologies

Sundial

Sundial otel, Fethiye, Turkey

A sundial is a device used since ancient times to estimate the time of day using the shadows cast by the sun. Research how a **sundial works**.

Explain, using diagrams, how the angle of the sun affects the length of the shadows.

Now, using this information, collaborate with a partner to **create** your own unique sundial. Take a series of measurements at certain times. Use the information to **find** the angle of the sun at various times of the day.

Assessment opportunities

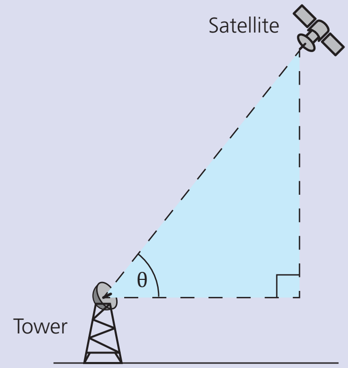
- In this activity you have practised skills that are assessed using Criterion C: Communicating, and Criterion D: Applying mathematics in real-life contexts. You might even choose to expand this activity to a Personal Project topic.

Where do geometric shapes occur around us?

Real-life problems: How do I find the hidden triangles?

It is very helpful to imagine the triangle 'hidden' in the diagram. First look for the lengths or measurements you have been given and mark them on the image. Remember to mark the right angle, both to show your communication and to self-check that you have correctly identified the relevant elements of the problem.

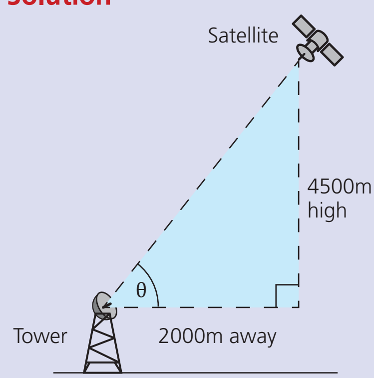
What if you have only words and no diagram? Don't let this worry you. **Identify** the dimensions/lengths in the question. **Sketch** a triangle and mark the right angle. **Match** the lengths (or angles) on the triangle. **Reread** the question to make sure you have correctly identified the information included. **Solve** as normal.



A tower has lost contact with a GPS satellite. The satellite is at 4500 metres over the Earth at a location 2000 metres from the tower.

At what angle should the tower point and scan to make contact again? Note the height of the tower is not included

Solution



$$\tan = \frac{O}{A}$$

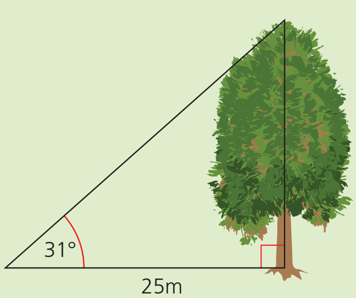
$$\tan\theta = \frac{4500}{2000}$$

$$\theta = \tan^{-1}\left(\frac{4500}{2000}\right)$$

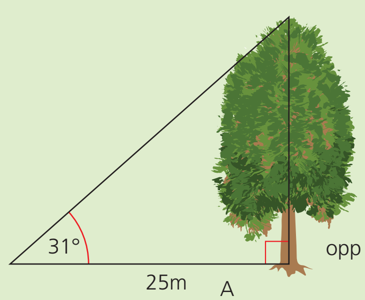
$$\theta = 66°$$

Example

Find the height of the tree.



Solution



$$\tan = \frac{O}{A}$$

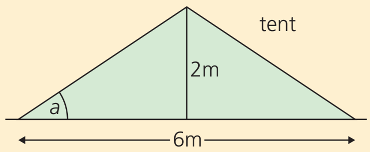
$$\tan\theta = \frac{\text{opp}}{25}$$

$$\text{opp} = 25 \times \tan 31$$

$$\text{opp} = 15\text{m}$$

The tree is 15 metres tall.

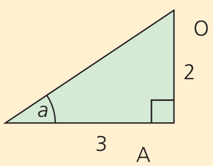
Worded problem: For a tent which is 2 m high and 6 m wide, what angle does the side of the tent make with the ground?



Solution

Find *a*.

Isolate triangle



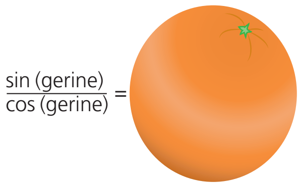
$\tan = \frac{O}{A}$

$\tan a = \frac{2}{3}$

$a = \tan^{-1}\frac{2}{3}$

$a = 33.7°$ to 3 s.f.

ACTIVITY: Can you get a tan in class?



- What does this actually mean?

Using what you know about trigonometric relationships and **substitution**, show why $\frac{\sin}{\cos}$ will result in a tan?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

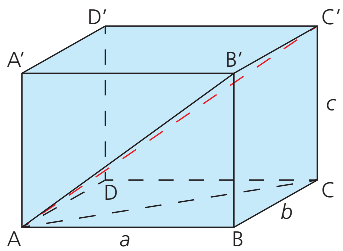
ACTIVITY: Will the relationships work in 3D models?

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations; Inquire in different contexts to gain a different perspective

Imagine you are trying to find a diagonal in a box. Or you are trying to find the longest length in a room for a laser beam security system. Or are base jumping off a building and want to make sure you know where you will land. Or are trying to calculate if your skis will go into a box that is definitely too short but might work on the diagonal.

Will Pythagoras help you in three dimensions?

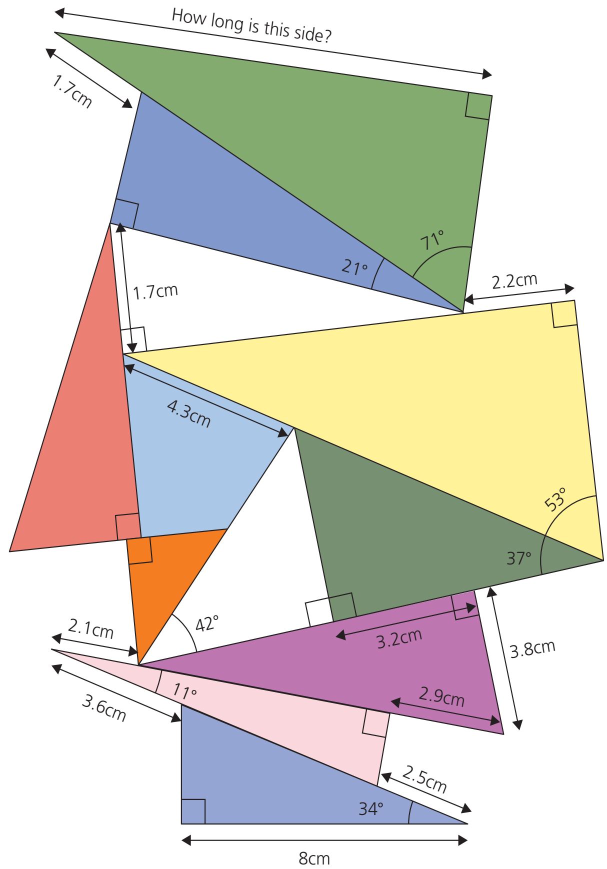


With a partner, **construct** a 3D model of a cuboid (or a 2D drawing to represent 3D space). **Determine** whether you could use Pythagoras' theorem to **find** the longest length (diagonal) inside the box if you know the dimensions of the box (cuboid).

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

ACTIVITY: Trigonometry pile up!



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Feel confident that you cover the whole framework with standard and extended mathematics included – with Extended clearly signposted

EXTENDED

Verify whether this calculation is correct for the diagram above!

ATL

- Self-management shills: Demonstrating resillience.

$$\frac{\sin 71 \times \sqrt{\left\{\sin 53 \times \left(4.3 + \left[\left(\left[\sqrt{\left[\left[\left(\frac{8}{\cos 34} - 2.5 + 3.6\right)\cos 11\right] - 2.1 + 2.9\right]^2 - 3.8^2}\right] - 3.2\right)\frac{\cos 42}{\sin 37}\right]\right) - 2.2\right\}^2 + 1.7^2}}{\cos 21}$$

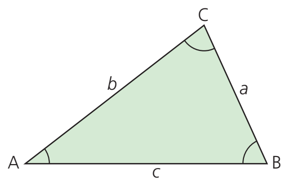
WHAT IS THE SINE RULE?

Now that you have mastered the lengths and sides of right-angled triangles of any size, we must begin to consider other non-right-angled triangles.

The sine rule

The first relationship which you can use to find unknowns is called the **sine rule**.

For any triangle ABC,



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Hint

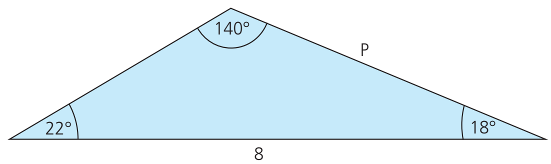
Notation

Remember that lowercase letters are used to denote sides and uppercase letters are given to angles. Note that the angles correspond in letters to their opposite sides.

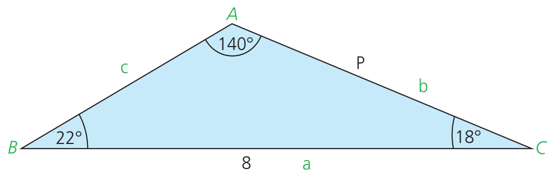
If you are given any two angles and one side of a triangle, you can use the sine rule to find the other sides. You can also use algebraic manipulation to find unknown sides. 2A + S or 2S + 1A problems. Often you will see these referred to as AAS (angle, angle, side) or SSA (side, side, angle) questions.

How much information do you need to solve a question using the sine rule?

Find P.



Solution



Write sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Identify sides and label.

Substitute $\dfrac{8}{\sin 140} = \dfrac{P}{\sin 22} = \dfrac{c}{\sin 18}$

We don't need all three versions to solve.

So take the first equation

$$\frac{8}{\sin 140} = \frac{P}{\sin 22}$$

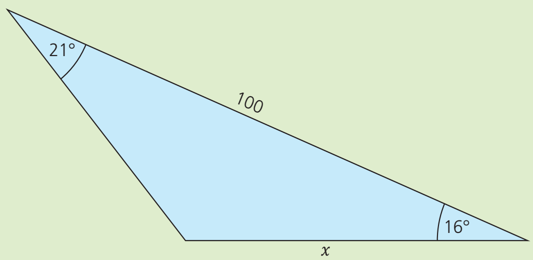
We want to isolate P so first cross-multiply

$$P \sin 140 = 8 \sin 22$$

$$P = \frac{8 \sin 22}{\sin 140} \longrightarrow \text{now plug into calculator}$$

$$P = 4.66$$

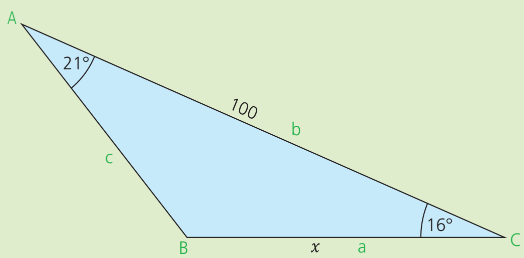
As you can see, we only need to identify the pairs of angles and sides we will use and solve from that single equation. Once you **identify** the correct pairs, all that is left to **solve** for the missing side or angle is to be careful and accurate in your algebraic manipulation.



Find x.

Solution

First **label** sides and angles.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

We are looking for x.

$$\frac{\sin A}{a} = \frac{\sin 143}{b}$$

$$\frac{\sin 21}{x} = \frac{\sin 143}{100}$$

We weren't given B but we know angles in a triangle add to 180°

so $B = 180 - 21 - 16$

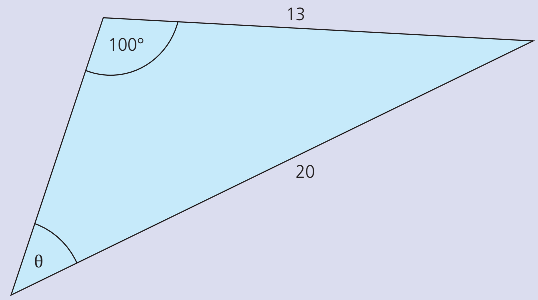
$B = 143°$

cross-multiply

$$x(\sin 143) = 100(\sin 21)$$

$$x = \frac{100 \sin 21}{\sin 143}$$

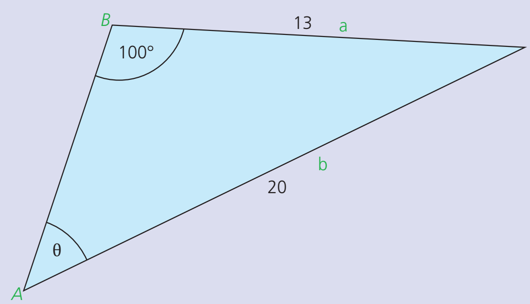
$$x = 59.5$$



Find θ.

Solution

Label sides and angles.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{13} = \frac{\sin 100}{20}$$

The order is not important as we will cross-multiply

$$20 \sin \theta = 13 \sin 100$$ cross-multiply

$$\sin \theta = \frac{13 \sin 100}{20}$$

$$\sin \theta = 0.6401250\ldots$$

To find the angle, use $\sin^{-1}$

$$\theta = \sin^{-1} 0.6401250\ldots$$

$$\theta \approx 39.8°$$

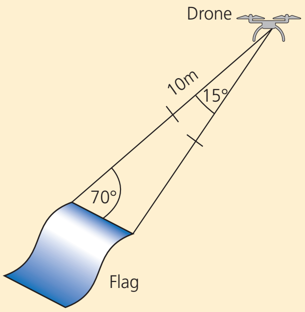
Hint

Don't forget the degree symbol when finding an angle.

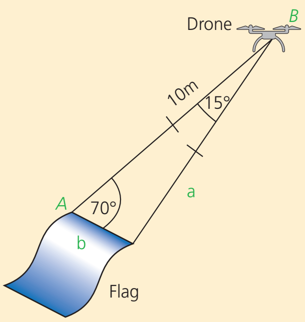
Drones are continuing to be used in new and varied ways all the time. Recently a protest using a drone and a flag disrupted an international football game and the game had to be stopped until the drone was brought down.

www.bbc.com/news/world-europe-29627615

Given the information in the diagram below, **find** the width of the flag being dragged by the drone.



Solution



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{\sin 70} = \frac{b}{\sin 15}$$

$$b \sin 70 = 10 \sin 15$$

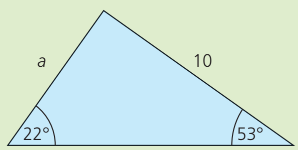
$$b = \frac{10 \sin 15}{\sin 70}$$

$$b = 2.75\,\text{m}$$

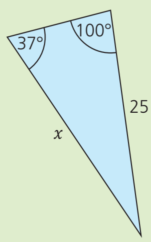
The flag was nearly 3 m wide!

PRACTICE QUESTIONS

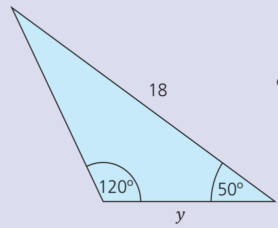
1 **Find** a.



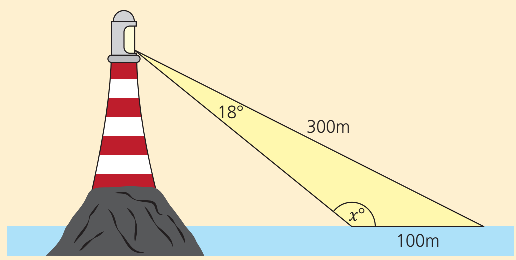
2 **Find** x.



3 **Find** y.



4 **Solve** for x.



Worked examples and practice questions are given in colour-coded boxes to show the level of difficulty

ACTIVITY: Would the sine rule work on a right-angled triangle as well?

ATL

- Collaboration skills: Understanding mathematical notation

Investigate if it works on right-angled triangles by **constructing** your own.

Test to see if both the sine rule and SOHCAHTOA gives the same answer.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

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Rita Bateson was, until very recently, the Curriculum Manager for MYP Mathematics and Sciences at the International Baccalaureate® (IB) and continues to be involved in curriculum review. She is an experienced teacher of MYP and DP Mathematics and Sciences, and is Head of Mathematics in her current school. She has taught in many international schools in Europe as well as North America. Her interests include overcoming mathematics anxiety in pupils and STEM education. She is also the co-author of *MYP by Concept 1-3 Mathematics*, with Irina Amlin.

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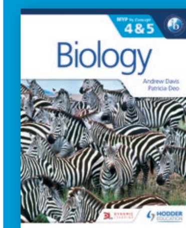

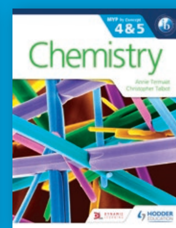

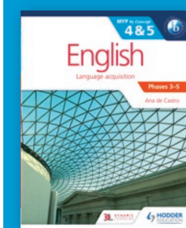

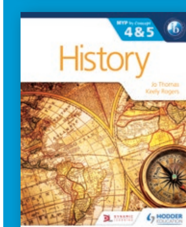

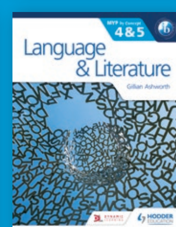

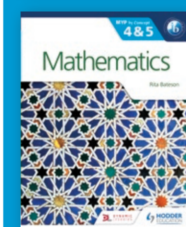

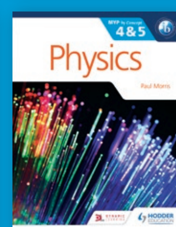

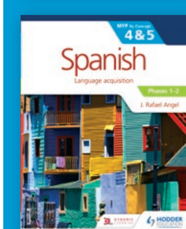

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